MATH 135 — QUIZ 5 — JAMES HOLLAND

2019-10-08

Question 1. Consider the function $f(x) = \frac{5}{x} + \frac{x}{5}$.

i. What are the critical points of f?

ii. What is the minimum value of f on the interval $[1, 10]$?

Solution .:.

i. The critical points of f are the values of x that make $f'(x) = 0$ or else have $f(x)$ defined but $f'(x)$ undefined. Firstly, $f'(x) = \frac{-5}{x^2} + \frac{1}{5}$ by the power rule. This is 0 iff $\frac{5}{x^2} = \frac{1}{5}$, which requires $25 = x^2$, meaning $x = 5$ or $x = -5$. $f'(x)$ is undefined iff $x = 0$, but since $f(x)$ is also undefined there, this isn't a critical point. So the critical points are 5 and -5.

ii. Since we are considering the interval $[1, 10]$, the only points we need to consider are 5, and the endpoints 1 and 10:

$$f(1) = \frac{5}{1} + \frac{1}{5} = 5.2$$
$$f(5) = \frac{5}{5} + \frac{5}{5} = 2$$
$$f(10) = \frac{5}{10} + \frac{10}{5} = 2.5.$$

Hence $f(5)$ is the minimum value of f on $[1, 10]$.

Question 2. Consider the function $f(x) = e^{\sin x}$.

i. Calculate $f'(x)$.

ii. What is the maximum value of f on $[0, \pi]$?

Solution .:.

i. Using the chain rule, $\frac{\mathrm{d}}{\mathrm{d}x} e^{\sin x} = e^{\sin x} \cdot \left(\frac{\mathrm{d}}{\mathrm{d}x} \cos x\right) = \cos(x) e^{\sin x} = f'(x)$.

ii. Intuitively, $f(x)$ is largest when $\sin(x)$ is largest, which happens when $\sin(x) = 1$, meaning the maximum value should be $e^1 = e$. We will prove this using the methods of calculus. First we must find the critical points of f. These are the values of x where $f'(x) = 0$ or $f'(x)$ is undefined. Since $f'(x) = \cos(x) \cdot e^{\sin(x)}$ is defined everywhere, x is a critical point whenever

$$f'(x) = \cos(x) e^{\sin x} = 0.$$

Since e^A is never 0 for any A, we can divide by it. So $f'(x) = 0$ whenever $\cos(x) = 0$. Within the interval $[0, \pi]$, this only happens at $x = \pi/2$. So to find the maximum value, we only need to compute $f(0)$, $f(\pi/2)$, and $f(\pi)$. We easily have $f(0) = f(\pi) = e^0 = 1$ whereas $f(\pi/2) = e > 1$. Hence e is the maximum value of f on $[0, \pi]$.